Birzeit University

Mathematics Department

Math 235 - First Exam

First Semester 2010/2011

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Question 1. (39%).Circle the most correct answer:

1. If f is differentiable function where:
   $f(2) = 3$, $f'(2) = 4$, then the derivative of: $h(x) = \frac{f(x)}{x^2-5}$ at $x = 2$ is:

   (a) -16
   (b) -19
   (c) 16
   (d) 19

   $\frac{(x^2-5)4 - (3)(2x)}{(x^2-5)^2} = \frac{(4-5)4-12}{1}$

2. The total profit and cost functions are given by: $p(x) = 3x^2-5x+6$, $c(x)=5-2x^2$.
   The marginal of the total revenue is given by:

   (a) $2x - 5$.
   (b) $-10x - 5$.
   (c) $10x - 5$.
   (d) $-10x + 5$.

   $P = R - C \Rightarrow R = -P - C$

3. suppose a radio manufacturer has the total cost function $c(x) = 3x + 400$, and the total revenue function $R(x) = 8x$. The number of units that must be sold in order to get profit of \$1000:

   (a) 120 units
   (b) 125 units
   (c) 200 units
   (d) 280 units

   $1000 = 8X - 3X - 400$
   $1400 = 5X$

4. The price-Demand equation for the production of calenders is given by: $x = 70 - 3p$, The marginal revenue at the level of production $x = 7$ is:

   (a) \$28.
   (b) $\$-3$.
   (c) $\$\frac{-1}{3}$.
   (d) \$18.

   $R = PX$
   $R = 70P - 3P^2$
   $R = 70 - 6P$

5. $\log \sqrt[3]{\frac{7}{2}} =$

(a) $\frac{1}{3}\log_3^2 - \frac{1}{3}\log_7^2$
(b) $\frac{1}{3}\log 2 - \frac{1}{3}\log 7$
(c) $\frac{1}{3}\log 2 - \log 7$
(d) $\frac{1}{3}\log 7 - \frac{1}{3}\log 2$

6. The points of discontinuity of $f(x) = \frac{x^2-3x-4}{x^2-5x}$

(a) $x = 0, x = 5$
(b) $x = 0, x = -5$
(c) $x = -1, x = 4$
(d) $x = 0 - 1, -5, 4$

$x^2 - 5x = 0$

7. suppose a radio manufacturer has the total cost function $c(x) = 3x + 40$, and the demand function is $p = x + 3$. The profit of Producing and selling 10 units is:

$R = x^2 + 3x$
$3x + 40 -$
$P = x^2 - 40$

(a) \$70.
(b) \$130.
(c) \$200.
(d) \$60.

8. If the cost function of a product is given by $c(x) = 3^{x+1} + 10x^2 + 200$ dollars ; then the fixed cost is:

(a) \$0.
(b) \$200.
(c) \$203.
(d) \$213.

9. If $R(x)$ is the total revenue function for a certain product, then the approximated revenue of the 13^{th} unit is:

(a) $R'(13)$
(b) $R(13)$
(c) $R'(12)$
(d) $R(13) - R(12)$

10. Solve for x: $2^{\log_2^{x^2}} - 4^{\log_4^{2x}} = e^{\ln 15}$

$4 - 16 = e^{\ln 15}$

(a) $x = 3$
(b) ~~x~~ $= 5$
(c) $x = 5, x = -3$
(d) $x = -5, x = 3$

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11. The domain of the function $f(x) = \frac{1}{\sqrt{1+x}}$ is

(a) $(-\infty, -1]$
(b) $(-\infty, -1)$
(c) $[-1, \infty)$
(d) $(-1, \infty)$

12. If $y = \frac{1}{\sqrt{x}}$, then $y' =$

$-\frac{\frac{1}{2} x^{-\frac{1}{2}}}{x}$

$= -\frac{1}{\quad}$

(a) $\frac{1}{2\sqrt[3]{x^2}}$
(b) $\frac{-1}{2\sqrt[3]{x^2}}$
(c) $\frac{1}{2\sqrt{x^3}}$
(d) $\frac{-1}{2\sqrt{x^3}}$

13. $\lim_{x \to 2} \frac{x^2-4}{|x-2|} =$

(a) -4
(b) 4
(c) 0
(d) DNE.

Question 2. *(12%)Find the following limits:*

1

1) $\lim_{x\to 3}\frac{x^2-9}{x^2-6x+9}$ $= \lim_{x\to 3} \frac{(\cancel{x-3})(x+3)}{\cancel{(x-3)}(\cancel{x-3})}$

$= \lim_{x\to 3} \frac{x+3}{x-3}$ ~~$= \lim_{x\to 3} \frac{x(1+\frac{3}{x})}{x(1-\frac{3}{x})}$~~

$= \lim_{x\to 3} \frac{\cancel{x}(1+\frac{3}{x})}{\cancel{x}(1-\frac{3}{x})}$

$= \lim_{x\to 3} -1 = -1$

3

2. $\lim_{x\to -1}\frac{2-2x^2}{x^2+x} + 3\sqrt{x^2+8}$

~~$= \lim_{x\to -1} \frac{x^2(\frac{2}{x^2}-2)}{x^2(1+\frac{1}{x})} + 3\sqrt{9}$~~

=

3. $\lim_{x\to -\infty}\frac{3x^4-2x^7}{x^4-2x-3}$ ~~$= \frac{3}{1} = 3$~~

3

Question 3. *(6%)*

3

1. *If* $y = 3 - \frac{2}{\sqrt{x}} - \frac{1}{\sqrt[5]{x^2}}$ *Find:* $\frac{dy}{dx}$

4

2. *If* $y = \frac{x^2-2x+3}{2x-3}$ *Find:* $\frac{dy}{dx}$ *at* $x = 2$

4

2

Question 4. (7%).

4

*a.*Find the value of an investment of \$10000 in 12 years if it earns an annual rate of 3.95% compounded continuously.

$A = Pe^{rt}$

2

$= 10000\, e^{3.95\% \cdot 12}$

$\Rightarrow = 10000 e^{0.47}$

$= 10000 \times 1.6$

$= 16000\$$

b. How many years will it take for \$1000 invested at 6% compounded quarterly to be worth \$1800?

4

2

$A = P\left(1 + \frac{r}{m}\right)^{mt}$

$1800 = 1000\left(1 + \frac{6\%}{4}\right)^{4t}$

$\cancel{1800 = 1.015^{4}}$

$1.8 = 1.015^{4t}$

$\cancel{\log 1.8 = \log 1.015^{4t}}$

$\Rightarrow 4t = \log_{1.015} 1.8$

$\Rightarrow \log_{1.015} 1.8 = 4t$

$t = \frac{\log_{1.015} 1.8}{4}$

9

Question 5. *(12%) The revenue (in dollars) from the sale of x infant car seats is given by:*

$$R(x) = 60x - 0.025x^2, \qquad 0 \le x \le 2400$$

Answer the following questions:

1. *Find the marginal average revenue function .*

$R'(x) = 60 - 0.05x$

2. *Find the average change in revenue if production is changed from 1000 car seats to 1050 car seats.*

Averge change $= \dfrac{R(1050) - R(1000)}{1050 - 1000} = \dfrac{35437.5 - 35000}{5}$

$= \dfrac{437.5}{5} = \boxed{87.5}$

3. *Find the rate of change of revenue at a production level 1000 car seats.*

rate of change $= R'(x) = R'(1000)$

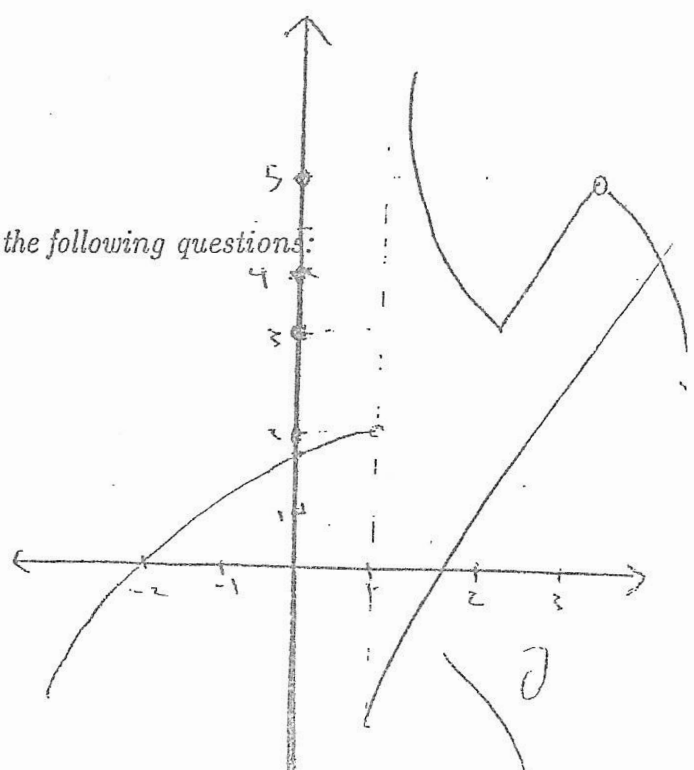
$= 60 - 0.05(1000) = \boxed{10}$

4. *what will the approximate change in revenue if the production is increased from 1000 car seats to 1050 car seats.*

approx change $= R'(1050)$

$\Rightarrow 60 - .05(1050) = \cancel{7.5}$

Question 6. *(6%) From the graph of f: Answer the following questions:*



1. $\lim_{x\to 3} f(x) =$ 5
2. $\lim_{x\to 1^+} f(x) =$ ∞
3. $\lim_{x\to \infty} f(x) =$ DNE
4. $\lim_{x\to -2} f(x) =$ 3
5. *Determine where the function is continuous?*

$(-\infty, 1) \cup (1, 3) \cup (3, \infty)$

Question 7. (18%).
Assume that the total cost function of a company's product is linear model, where the fixed cost is \$7, and the total cost of producing 10 units is \$67.
Also, the demand function of the product is given by $p(x) = 14 - x$, where p is the price per unit in dollars, and x is the number of units demanded.
Find the following:

a) The total cost function.
b) The total revenue function.
c) Break-Even points.
d) The approximated profit of producing and selling the 11^{th} unit.
e) The marginal of average cost function.
f) The rate of change of marginal revenue at $x = 3$.

fixed cost = 7, 10 ⟶ 67, $P(X) = 14 - X$

$R = P.X = 14X - X^2$

a) $C(X) = C(10) = 67$

$C(X) = 10X - 3$

3 b) $R = P.X = 14X - X^2$

c) $R = C$

$14X - X^2 = 10X - 3$

$4X - X^2 + 3 = 0$

d) $P = R - C$

$= 4X - X^2 - 3$

$P' = X - 2X$

e) $C'(X) = 10$

4